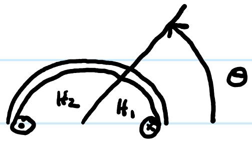
2018-10-17-1

Continuing Examing Rotational System:

$$H_1 = \frac{N_s i_s - N_r i_r}{2g} = -H_3 \qquad V_s = \frac{d\lambda_s}{dt}$$

$$H_2 = \frac{N_s i_s + N_r i_r}{2g} = -H_4 \qquad V_r = \frac{d\lambda_r}{dt}$$

Stator Flux linkage:



$$\phi_s = \int_s \underline{B}\cdot\hat{n}\,dA = \int_0^l\int_0^\pi B\,R\,d\gamma\,dl = \int_0^\pi B\,l\,R\,d\gamma = \int_0^\theta \mu_0 H_1\, lR\,d\gamma + \int_\theta^\pi \mu_0 H_2\, lR\,d\gamma$$

$$\phi_s = \mu_0 lR\theta H_1 + \mu_0 lR(\pi-\theta)H_2$$

$$= \mu_0 lR\theta\left(\frac{N_s i_s - N_r i_r}{2g}\right) + \mu_0 lR\pi\left(\frac{N_s i_s + N_r i_r}{2g}\right) - \mu_0 lR\theta\left(\frac{N_s i_s + N_r i_r}{2g}\right)$$

$$\phi_s = \mu_0 lR\pi\left(\frac{N_s i_s + N_r i_r}{2g}\right) - \mu_0 lR\theta\left(\frac{2N_r i_r}{2g}\right)$$

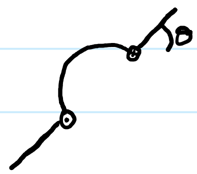
$$\phi_s = \left(\frac{\mu_0 lR\pi N_s}{2g}\right)i_s + \left(\frac{\mu_0 lR\pi N_r}{2g}\left[1-\frac{2\theta}{\pi}\right]\right)i_r$$

$$\lambda_s = N_s\phi_s$$

$$\boxed{\begin{aligned}&\lambda_s = \left(\frac{\mu_0 lR\pi N_s^2}{2g}\right)i_s + \left(\frac{\mu_0 lR\pi N_s N_r}{2g}\left[1-\frac{2\theta}{\pi}\right]\right)i_r\\ &\lambda_s = L_s i_s + L_m(\theta)i_r\\ &0<\theta<\pi\end{aligned}}$$

2018-10-17-2

Rotor flux Linkage:



$$\phi_r = \int_s \underline{B}\cdot\hat{n}\,dA = \int_\theta^{\pi+\theta}\int_0^{l} B\,dl\,R\,d\gamma = \int_\theta^{\pi+\theta} B\,l\,R\,d\gamma = \int_\theta^{\pi}\mu_0 H_2\,l\,R\,d\gamma + \int_\pi^{\pi+\theta}\mu_0 H_3\,l\,R\,d\gamma$$

$$\Theta_r = \mu_0 l R(\pi-\theta)H_2 + \mu_0 l R\theta H_3$$

$$= \mu_0 l R\pi\left(\frac{N_s i_s + N_r i_r}{2g}\right) - \mu_0 l R\theta\left(\frac{N_s i_s + N_r i_r}{2g}\right) + \mu_0 l R\theta\left(\frac{N_r i_r - N_s i_s}{2g}\right)$$

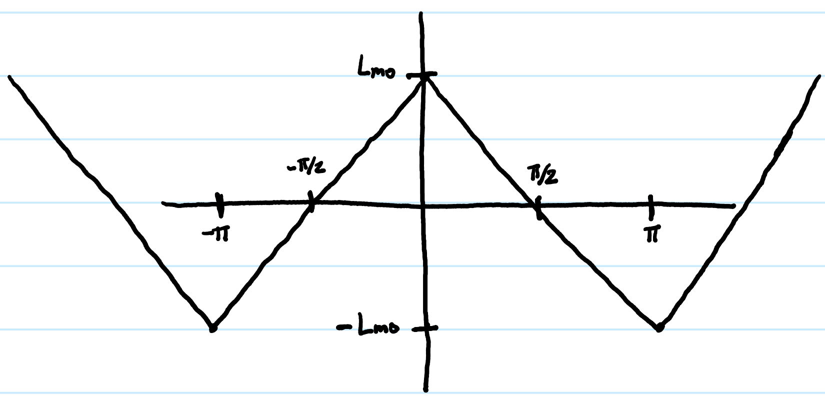
$$= \mu_0 l R\pi\left(\frac{N_s i_s + N_r i_r}{2g}\right) - \mu_0 l R\theta\left(\frac{2N_s i_s}{2g}\right)$$

$$\phi_r = \left(\frac{\mu_0 l R\pi N_s}{2g}\left[1-\frac{2\theta}{\pi}\right]\right) i_s + \left(\frac{\mu_0 l R\pi N_r}{2g}\right) i_r$$

$\lambda_r = N_r \phi_r$

$$\boxed{\begin{aligned} &\lambda_r = \left(\frac{\mu_0 R l \pi N_s N_r}{2g}\left[1-\frac{2\theta}{\pi}\right]\right) i_s + \left(\frac{\mu_0 l R\pi N_r^2}{2g}\right) i_r \\ &\lambda_r = L_m(\theta) i_s + L_r i_r \\ &0 < \theta < \pi \end{aligned}}$$

$$L_{mo} = \frac{\mu_0 l R\pi N_s N_r}{2g}$$



* for $-\pi < \theta < 0$, simply replace θ by $-\theta$ so that $L_m(\theta) = L_{mo}\left(1+\frac{2\theta}{\pi}\right)$

* Can create system (using additional slots and windings) where

$$L_m(\theta) = M\cos(\theta)$$

$$V_s = \frac{d\lambda_s}{dt} = \frac{d}{dt}\left(L_s i_s\right) + \frac{d}{dt}\left(M\cos(\theta)\, i_r\right) \Rightarrow \boxed{V_s = L_s\frac{di_s}{dt} + M\cos(\theta)\frac{di_r}{dt} - M\sin(\theta)\frac{d\theta}{dt}\, i_r}$$

$$V_r = \frac{d\lambda_r}{dt} = \frac{d}{dt}\left(M\cos(\theta)\, i_s\right) + \frac{d}{dt}\left(L_r i_r\right) \Rightarrow \boxed{V_r = L_r\frac{di_r}{dt} + M\cos(\theta)\frac{di_s}{dt} - M\sin(\theta)\frac{d\theta}{dt}\, i_s}$$

<u>Electrically linear system:</u> Relating i to λ

i) Single coil

$$\lambda = L(x)\, i$$

$$i = \frac{\lambda}{L(x)}$$

ii) Multiple coils

$$\lambda_s = L_s i_s + L_m(\theta) i_r$$

$$\lambda_r = L_m(\theta) i_s + L_r i_r$$

$$\Rightarrow \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_s & L_m(\theta) \\ L_m(\theta) & L_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

$$\underline{\lambda} = \underline{\underline{L}}\, \underline{i}$$

$$\underline{i} = \underline{\underline{L}}^{-1} \underline{\lambda}$$

$$i_s = \frac{1}{L_s L_r - L_m^2(\theta)}\left(L_r \lambda_s - L_m(\theta)\lambda_r\right)$$

$$i_r = \frac{1}{L_s L_r - L_m^2(\theta)}\left(-L_m(\theta)\lambda_s + L_s\lambda_r\right)$$

* Using either i or λ as independent variable is allowed
* Using i as independent variable is much easier.